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A Novel Analytical Method for Selective Harmonic Elimination Problems in Five-Level Converters

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Abstract

Multilevel converters have attracted a lot of attention in recent years. The efficiency parameters of a multilevel converter such as the switching losses and total harmonic distortion (THD) mainly depend on the modulation strategy used to control the converter. Among all of the modulation techniques, the selective harmonic elimination (SHE) method is particularly suitable for high-power applications due to its low switching frequency and high quality output voltage. This paper proposes a new expression for the SHE problem in five-level converters. Based on this new expression, a simple analytical method is introduced to determine the feasible modulation index intervals and to calculate the exact value of the switching angles. For each selected harmonic, this method presents three-level or five-level waveforms according to the value of the modulation index. Furthermore, a flowchart is proposed for the real-time implementation of this analytical method, which can be performed by a simple processor and without the need for a lookup table. The performance of the proposed algorithm is evaluated with several simulation and experimental results for a single phase five-level diode-clamped inverter.

Key words: selective harmonic elimination (SHE), multilevel converters, diode-clamped inverter, modulation techniques, real time implementation

I. INTRODUCTION

Nowadays, the use of multilevel converters is popular in medium-voltage medium-power applications such as medium-voltage drives [1], flexible AC transmission systems (FACTS) [2], and grid-connected photovoltaic systems [3]. Compared with two level converters, multilevel converters have more benefits. For instance, they can work at lower switching frequencies, they have lower switching losses and lower switching stresses across the switching devices [4-6].

Several modulation and control techniques have been proposed for multilevel converters including sinusoidal PWM (SPWM) [7-9], space vector modulation (SVM) [10-12], and selective harmonic elimination (SHE) [13].

The switching frequency must be kept low in high power and efficient applications because the switching losses are directly proportional to the switching frequency. The most popular and simple low frequency switching modulation technique for multilevel converters is selective harmonic elimination (SHE).

The aim of this method is to determine the switching angles in a manner that eliminates low order harmonics. A fundamental problem with the SHE method obtaining the arithmetic solution of nonlinear transcendental equations which contain trigonometric terms and naturally present multiple solutions [13-16].

Several algorithms have been suggested in the literature to solve SHE trigonometric equations. In [17,18], iterative numerical techniques such as the Newton–Raphson method have been implemented to solve SHE equations while producing only one solution set. Furthermore, these techniques may get trapped in a local optima and require a proper initial guess. In [19,20], the resultant theory has been suggested to solve SHE equations. In this technique, SHE transcendental equations are converted into an equivalent set of polynomial equations. The order of the polynomials is proportional to the number of switching angles. Hence, increasing the number of switching angles makes the degree of the polynomial extremely large. A stochastic optimization technique based on a Genetic Algorithm (GA) was proposed for computing switching angles in [14,21]. Proper selection of effective parameters such as population size, mutation rate, etc. is necessary. Consequently, its implementation is difficult for a high number of levels [13,14]. Another optimization technique is the Particle Swarm Optimization (PSO) approach which was developed to deal with the SHE problem in [22,23]. In comparison with a GA, this technique has the benefit of a high rate of convergence and precision. However, both techniques may become trapped in local optima. The bee algorithm [24] and the ant colony algorithm [25] are other optimization techniques that have been used to optimize the cost functions for providing solutions to the multilevel SHE problem.

Regretfully, there is no analytical method to solve SHE transcendental equations for n-level converters. However, a new analytical method has recently been introduced in [26] for five-level converters which is able to calculate the exact amount of switching angles for the elimination of each selected harmonic. The authors assume that the scheme of the output voltage waveform is always a staircase similar to Fig. 1. However, there is another possible switching scheme which may work over different ranges of the modulation index.

In this paper, a new expression for the SHE problem in five-level converters has been introduced. It can explain three-level and five-level waveforms with two switching angles through a single formulation. Furthermore, the analytical procedure for determining feasible modulation index intervals and all possible sets of solutions is presented based on the proposed expression. Using this technique, the exact value of switching angles is easily calculated. For the real-time implementation of this method in microcontrollers or DSPs, a simple flowchart is proposed which does not need a lookup table.

II. Standard Problem Formulation for the SHE Technique in Five-Level Converters

Fig. 1 shows a staircase voltage waveform synthesized by a $(2N+1)$ level converter. Using the Fourier series, the stepped multilevel voltage waveform of Fig. 1 can be expressed by (1):

$$v_{Out}(\omega t)=\sum_{n=1}^{\infty} V_n^{Out}\, sin(n\omega t) \quad (1)$$

where n is the harmonic number, and V_n^{Out} represents the amplitude of the harmonic order n.

Due to odd quarter wave symmetry, the amplitude for all of the even order harmonics is zero. Since this waveform decomposes to N quasi square waveforms, the amplitude of odd order harmonics is calculated by (2).

$$V_n^{Out}=\frac{4V_{DC}}{n\pi}\sum_{i=1}^{N} cos(n\theta_i) \quad , \quad n=1,3,5,7,... \quad (2)$$

The following restrictions must be satisfied by the switching angles [14].

$$0\le\theta_1\le\theta_2\le\cdots\le\theta_N\le\pi/2 \quad (3)$$

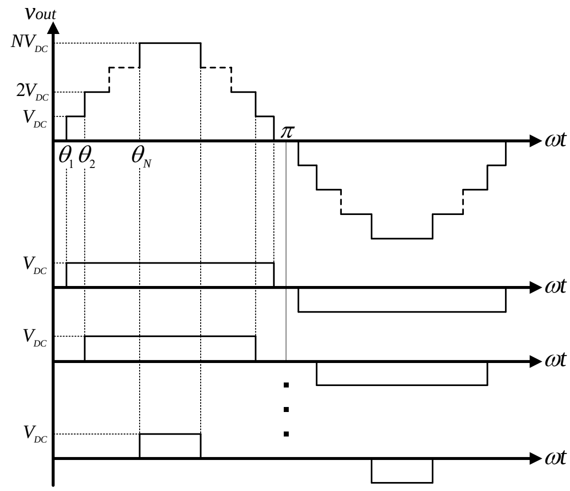

Fig. 1. The output voltage waveform of an n-level converter.

The objective of the SHE problem is to determine the series of switching angles $(\theta_1-\theta_N)$ so that the specified low order harmonics are eliminated. At the same time, the amplitude of the fundamental harmonic becomes equal to the desired value.

By defining the modulation index M in equation (4), the SHE problem can be formulated by equations (5) where the number of equations should be equal to or less than N [13,14].

$$M=\frac{V_1^{Out}}{4NV_{DC}/\pi} \quad (0\le M\le 1) \quad (4)$$

$$\begin{cases}\sum_{i=1}^{N} cos(\theta_i)=NM \\ \sum_{i=1}^{N} cos(n\theta_i)=0 \qquad n=3,5,7,...\end{cases} \quad (5)$$

For example, for a five-level converter, the output voltage waveform is composed of two quasi square waveforms as shown in Fig. 2. Due to having two switching angles $(N=2)$, the SHE equations should be two or less. In other words, the switching angles (θ_1,θ_2) should be determined to attain the desired amplitude of the fundamental harmonic and to suppress the one selected harmonic. The following equations explain the SHE problem for five-level converters where n is the harmonic order which is going to be eliminated.

$$\begin{cases}cos(\theta_1)+cos(\theta_2)=2M \quad , \quad M=\frac{V_1^{Out}}{8V_{DC}/\pi} \\ cos(n\theta_1)+cos(n\theta_2)=0\end{cases} \quad (6)$$

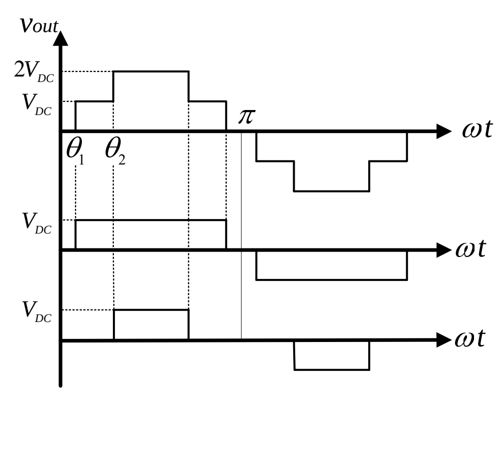

Fig. 2. Output voltage waveform of a five-level converter.

III. New Expression for the SHE Problem in Five-Level Converters

In this section, a new expression is proposed for the SHE problem in five-level converters. Consider a quasi-square waveform named base waveform with a switching angle α. As shown in Fig. 3, the output voltage of a five-level converter is obtained by subtracting two base waveforms with different phases. In other words, instead of using two quasi-square waveforms with different switching angles (θ_1 and θ_2), as shown in Fig. 2, the output waveform can be provided using two quasi-square waveforms with the same switching angles and different phases.

The fundamental harmonic amplitude of both base waveforms (V_1^{Base}) are determined by (7) which can be adjusted by α.

$$V_1^{Base} = \frac{4V_{DC}}{\pi} cos(\alpha) \tag{7}$$

In addition, the n^{th} order harmonic amplitude of the base waveforms (V_n^{Base}) can be determined by (8).

$$V_n^{Base} = \frac{4V_{DC}}{n\pi} cos(n\alpha) \tag{8}$$

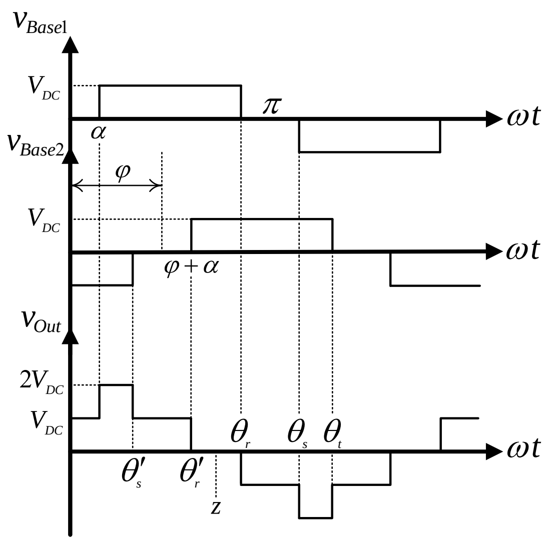

Fig. 3. Output voltage waveform of a five-level converter (new expression, five-level waveform).

The fundamental harmonic amplitude of the output waveform (V_1^{Out}) can be calculated by subtracting the fundamental harmonic waveform of the base waveforms as follows:

$$v_1^{Out}(t) = V_1^{Base} sin(\omega t) - V_1^{Base} sin(\omega t - \varphi)$$

$$\Rightarrow\ v_1^{Out}(t) = V_1^{Base}[2\,sin(\varphi/2)\,cos(\omega t - \varphi/2)]$$

$$\Rightarrow\ V_1^{Out} = \left[\frac{4V_{DC}}{\pi} cos(\alpha)\right][2\,sin(\varphi/2)]$$

$$\Rightarrow\ V_1^{Out} = \frac{8V_{DC}}{\pi} cos(\alpha)\,sin(\varphi/2)$$

Using equation (4):

$$cos(\alpha)\,sin(\varphi/2) = M \tag{9}$$

Equation (9) shows the dependency between the fundamental harmonic amplitude of the output waveform to the switching angle α and the phase difference φ.

In a similar way, the n^{th} order harmonic amplitude of the output waveform can be written as:

$$v_n^{Out}(t) = V_n^{Base} sin(n\omega t) - V_n^{Base} sin[n(\omega t - \varphi)]$$

$$\Rightarrow v_n^{Out}(t) = V_n^{Base}[2\,sin(n\,\varphi/2)\,cos(n\omega t - n\varphi/2)]$$

$$\Rightarrow\ V_n^{Out} = \left[\frac{4V_{DC}}{n\pi} cos(n\alpha)\right][2sin(n\varphi/2)]$$

$$\Rightarrow\ V_n^{Out} = \frac{8V_{DC}}{n\pi} cos(n\alpha)\,sin(n\varphi/2) \tag{10}$$

Equation (10) indicates that the n^{th} order harmonic amplitude of the output voltage can also be adjusted by the switching angle α and phase difference φ.

According to (9) and (10), the SHE problem for five-level converters is redefined as follows:

$$\begin{cases} \cos(\alpha)\sin(\varphi/2) = M \\ \cos(n\alpha)\sin(n\varphi/2) = 0 \end{cases} \tag{11}$$

where the first equation defines the fundamental voltage magnitude and the second one guarantees to completely eliminate the n^{th} order harmonic [27].

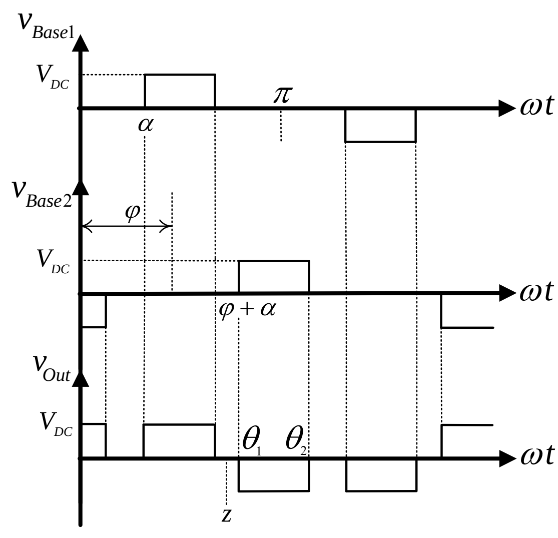

Fig. 4. Output voltage waveform of a five-level converter (new expression, three-level waveform).

This expression generates the usual staircase five-level waveform and systematically produces a three-level waveform if $\alpha > \varphi/2$ as shown in Fig. 4. It is noticeable that the SHE problem for both five-level and three-level waveforms is (11). Therefore, this new expression is more generalized than the usual SHE formulation.

IV. Analytical Procedure for Modulation Index Intervals

Solving equation (11) is simpler when compared to solving the standard SHE problem. To eliminate the desired n^{th} order harmonic, it is enough to choose the phase difference φ according to (12).

$$\sin(n\varphi/2)=0 \;\Rightarrow\; n\varphi/2=k\pi \;\Rightarrow\; \varphi=2k\pi/n \quad (12)$$

where k is a positive integer and all results less than π are acceptable. For attaining a desired amplitude of the fundamental harmonic, it is adequate to calculate the switching angle α by (13).

$$cos(\alpha)\,sin(\varphi/2)=M \;\Rightarrow\; \alpha=cos^{-1}\left(\frac{M}{sin(\varphi/2)}\right) \quad (13)$$

According to (13), to obtain a real value for the switching angle α, the modulation index must observe the following restriction.

$$0\le M\le sin(\varphi/2) \quad (14)$$

Based on the value of α and φ, the output waveform may have three or five levels. The border of five-level and three-level waveforms is determined by $\alpha=\varphi/2$. Using (9), the border modulation index (M_{Border}) is calculated by (15).

$$M_{Border}=cos(\varphi/2)\,sin(\varphi/2) \quad (15)$$

Consequently, equation (16) shows the feasible modulation index intervals for three-level and five-level output waveforms for an acceptable phase difference φ.

$$\begin{cases}0\le M\le cos(\varphi/2)\,sin(\varphi/2)\\ \qquad\Rightarrow\ \text{three}-\text{level output waveform}\\ cos(\varphi/2)\,sin(\varphi/2)<M\le sin(\varphi/2)\\ \qquad\Rightarrow\ \text{five}-\text{level output waveform}\end{cases} \quad (16)$$

In fact, this technique systemically proposes a three-level output waveform scheme for a small value of the modulation index contrary to the usual staircase five-level waveform. Using this technique, the minimum value of the modulation index is zero for all conditions [27]. The calculation of the feasible modulation index intervals for the third, fifth, and seventh harmonics are discussed in detail.

A. Modulation index intervals for the elimination of the third harmonic (n=3)

Using (12).

$$n=3 \;\Rightarrow\; \varphi=2k\pi/3$$

The only acceptable answer is $\varphi=2\pi/3$ (by considering that $k=1$). Under this condition, the feasible modulation index interval for three-level and five-level output waveforms is obtained as follows:

$$\begin{cases}0\le M\le cos(\pi/3)\,sin(\pi/3)\\ \qquad\Rightarrow\ \text{three}-\text{level output waveform}\\ cos(\pi/3)\,sin(\pi/3)<M\le sin(\pi/3)\\ \qquad\Rightarrow\ \text{five}-\text{level output waveform}\end{cases}$$

$$\Rightarrow\begin{cases}0\le M\le 0.4330\\ \qquad\Rightarrow\ \text{three}-\text{level output waveform}\\ 0.4330<M\le 0.8660\\ \qquad\Rightarrow\ \text{five}-\text{level output waveform}\end{cases}$$

B. Modulation index intervals for the elimination of the fifth harmonic (n=5)

$$n=5 \;\Rightarrow\; \varphi=2k\pi/5$$

There are two acceptable answers. Therefore, two different switching angles are achieved. Using (16), the feasible modulation index interval for each answer is demonstrated as follows:

$$k=1 \;\Rightarrow\; \varphi'=2\pi/5$$

$$\Rightarrow\begin{cases}0\le M\le 0.4755\\ \qquad\Rightarrow\ \text{three}-\text{level output waveform}\\ 0.4755<M\le 0.5878\\ \qquad\Rightarrow\ \text{five}-\text{level output waveform}\end{cases}$$

$$k=2 \;\Rightarrow\; \varphi''=4\pi/5$$

$$\Rightarrow\begin{cases}0\le M\le 0.2939\\ \qquad\Rightarrow\ \text{three}-\text{level output waveform}\\ 0.2939<M\le 0.9511\\ \qquad\Rightarrow\ \text{five}-\text{level output waveform}\end{cases}$$

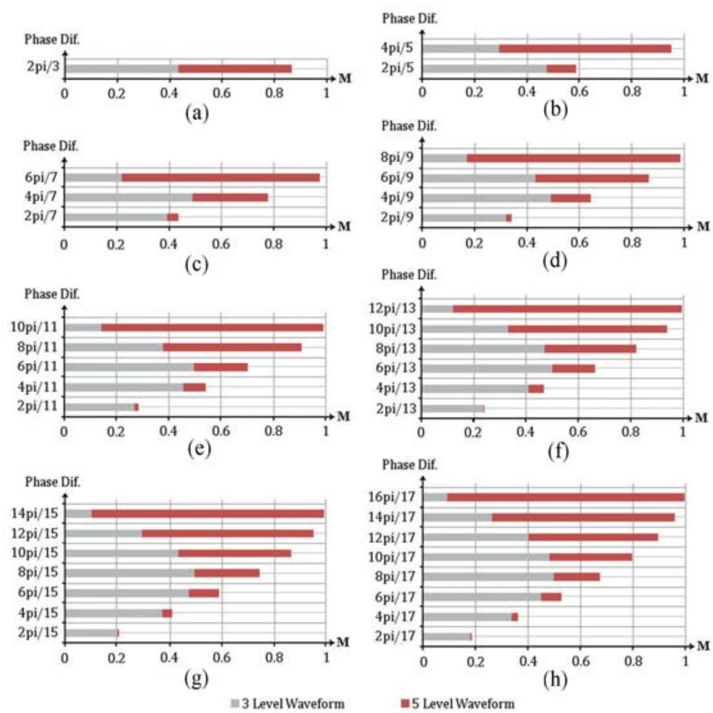

Fig. 5. Acceptable phase differences and their modulation index intervals for: (a) third harmonic, (b) fifth harmonic, (c) seventh harmonic, (d) ninth harmonic, (e) eleventh harmonic, (f) thirteenth harmonic, (g) fifteenth harmonic, (h) seventeenth harmonic.

C. Modulation index intervals for the elimination of the seventh harmonic (n=7)

$$n=7 \;\Rightarrow\; \varphi=2k\pi/7$$

Under this condition, there exist three different acceptable answers. Using (16), the feasible modulation index interval for each answer is calculated as follows:

$$k=1 \;\Rightarrow\; \varphi'=2\pi/7$$

$$\Rightarrow \begin{cases} 0 \le M \le 0.3909 \\ \qquad \Rightarrow \text{ three} - \text{level output waveform} \\ 0.3909 < M \le 0.4339 \\ \qquad \Rightarrow \text{ five} - \text{level output waveform} \end{cases}$$

$k = 2 \;\Rightarrow\; \varphi'' = 4\pi/7$

$$\Rightarrow \begin{cases} 0 \le M \le 0.4875 \\ \qquad \Rightarrow \text{ three} - \text{level output waveform} \\ 0.4875 < M \le 0.7818 \\ \qquad \Rightarrow \text{ five} - \text{level output waveform} \end{cases}$$

$k = 3 \;\Rightarrow\; \varphi''' = 6\pi/7$

$$\Rightarrow \begin{cases} 0 \le M \le 0.2169 \\ \qquad \Rightarrow \text{ three} - \text{level output waveform} \\ 0.2169 < M \le 0.9749 \\ \qquad \Rightarrow \text{ five} - \text{level output waveform} \end{cases}$$

Modulation index intervals for the elimination of the other odd harmonics can be determined by using this technique. Fig. 5 shows all of the acceptable phase differences and their modulation index intervals for each harmonic.

V. Analytical Procedure for Calculating Switching Angles

Using (12) and (13), it is easy to calculate $(\alpha \text{ and } \varphi)$ for eliminating each selected harmonic for any desired modulation index. The unknown parameters in the standard SHE problem are $(\theta_1 \text{ and } \theta_2)$ which can be calculated in accordance with $(\alpha \text{ and } \varphi)$. As shown in Fig. 3, the switching angles $(\theta_1 \text{ and } \theta_2)$ are defined relative to the new origin (point z) whose coordinate is calculated by (17).

$$z = \pi/2 + \varphi/2 \tag{17}$$

Now, it is easy to find the output switching angles:

$$\theta_r = (\pi - \alpha) - z = \pi/2 - \alpha - \varphi/2$$

$$\acute{\theta}_r = (\varphi + \alpha) - z = -\pi/2 + \alpha + \varphi/2$$

$$\theta_s = (\pi + \alpha) - z = \pi/2 + \alpha - \varphi/2$$

$$\acute{\theta}_s = (\varphi - \alpha) - z = -\pi/2 - \alpha + \varphi/2$$

$$\theta_t = (\varphi + \pi - \alpha) - z = \pi/2 - \alpha + \varphi/2$$

Notice that $\acute{\theta}_r$ is opposite θ_r and that $\acute{\theta}_s$ is opposite θ_s. Therefore, all of the switching angles in Fig. 3 are formulated as follows:

$$\begin{cases} \theta_r = |\pi/2 - \alpha - \varphi/2| \\ \theta_s = |\pi/2 + \alpha - \varphi/2| \\ \theta_t = \pi/2 - \alpha + \varphi/2 \end{cases} \tag{18}$$

θ_1 and θ_2 are the ones that are less than $\pi/2$.

Fig. 6 shows a flowchart of this new approach to calculate the switching angles of the SHE problem in five-level converters. This flowchart can be implemented in real time using a simple processor or DSP without the need for a lookup table. It is more general and easier than other analytical methods such as the one proposed in [24]. It is noticeable that all of the equations in this flowchart are polynomial except for (13) which can be easily calculated using Taylor series. This makes the proposed algorithm simple and fast.

In brief, the advantages of the proposed method are as follows:

- Determining the exact values of switching angles.
- Avoiding the need to solve complex trigonometric equations using a novel analytical method.
- Determining all possible sets of solutions and the exact boundaries for all valid modulation index intervals.
- Systematically providing three-level schemes to extend the modulation index intervals.
- Easy for real-time implementation.

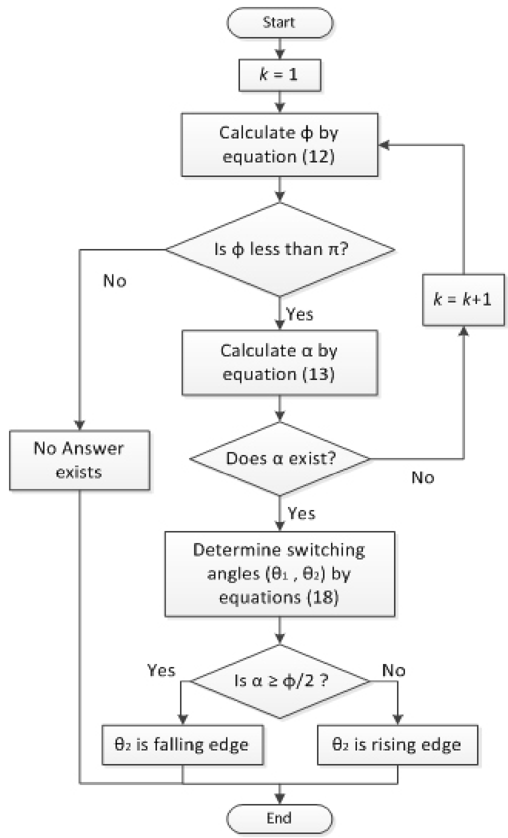

Fig. 6. Flowchart of the proposed algorithm.

VI. Simulation Results

To investigate the validity of the solutions obtained by the new formulation proposed in this paper, a five-level diode-clamped inverter is simulated in MATLAB/SIMULINK. Fig. 7 shows one leg of a five-level diode-clamped inverter. The characteristics of the converter are as follows:

- Total DC input voltage is 6 kV ($V_{DC} = 1.5\ kV$).
- Output frequency is 50Hz.

Several cases are examined in which the selected harmonic is eliminated and the desired fundamental amplitude is achieved.

Case 1: Elimination of the third harmonic for $\boldsymbol{M=0.8}$

As shown in Fig. 5(a), only one solution exists and its output waveform has five levels. The output voltage waveform and its FFT spectrum are shown in Fig. 8.

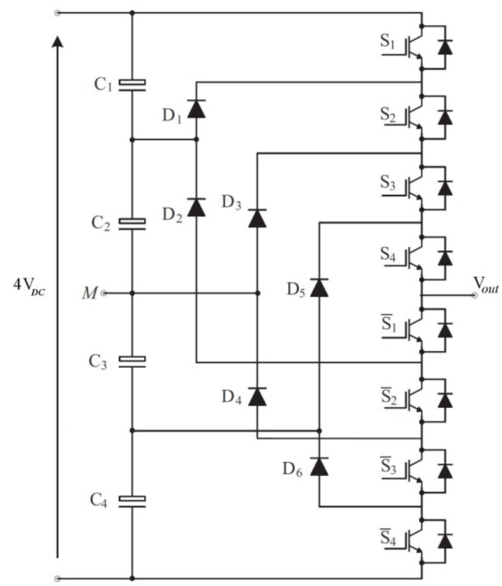

Fig. 7. One leg of a five-level diode-clamped inverter.

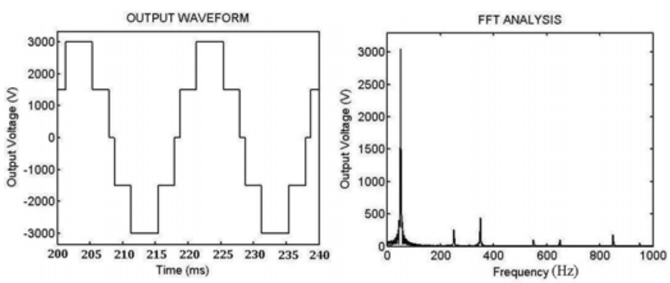

Fig. 8. Elimination of the third harmonic for *M=0.8*, using phase a difference of $\varphi=2\pi/3$ (Simulation results).

Case 2: Elimination of the seventh harmonic for $\boldsymbol{M=0.46}$

According to Fig. 5(c), two solutions exist, one for $\varphi=4\pi/7$ with a three-level output waveform, and the other for $\varphi=6\pi/7$ with a five-level output waveform. Output voltage waveforms and their FFT spectrums are shown in Fig. 9.

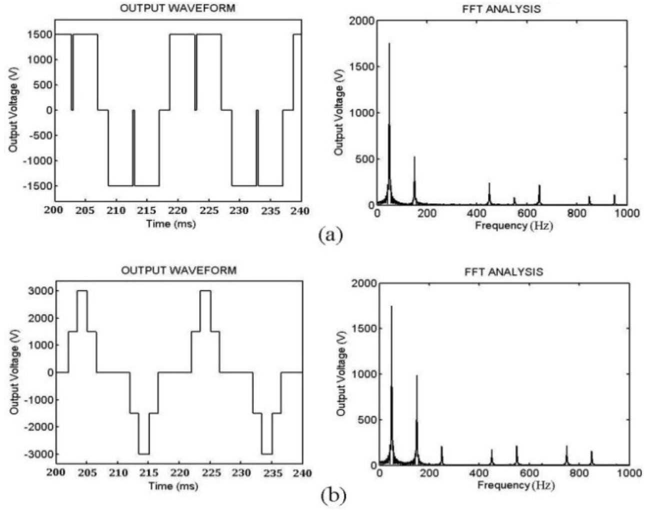

Fig. 9. Elimination of the seventh harmonic for *M=0.46*: (a) $\varphi=4\pi/7$, (b) $\varphi=6\pi/7$ (Simulation results).

Case 3: Elimination of the thirteenth harmonic for $\boldsymbol{M=0.9}$

Fig. 10 shows simulation results of this condition. There exist two acceptable answers for $\varphi=10\pi/13$ and $\varphi=12\pi/13$, both of which have a five-level output waveform.

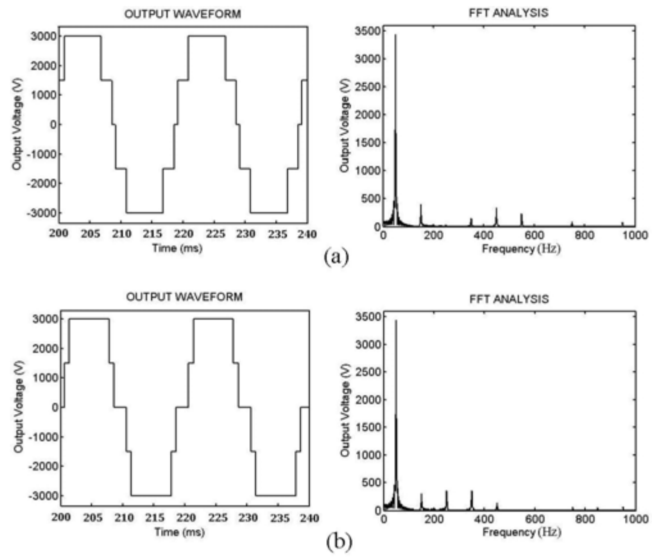

Fig. 10. Elimination of the thirteenth harmonic for *M=0.9*: (a) $\varphi=10\pi/13$, (b) $\varphi=12\pi/13$ (Simulation results).

Case 4: Elimination of the seventeenth harmonic for $\boldsymbol{M=0.2}$

According to Fig. 5(h), there exist seven different solutions. For instance, Fig. 11 shows simulation results for $\varphi=4\pi/17$ and $\varphi=16\pi/17$.

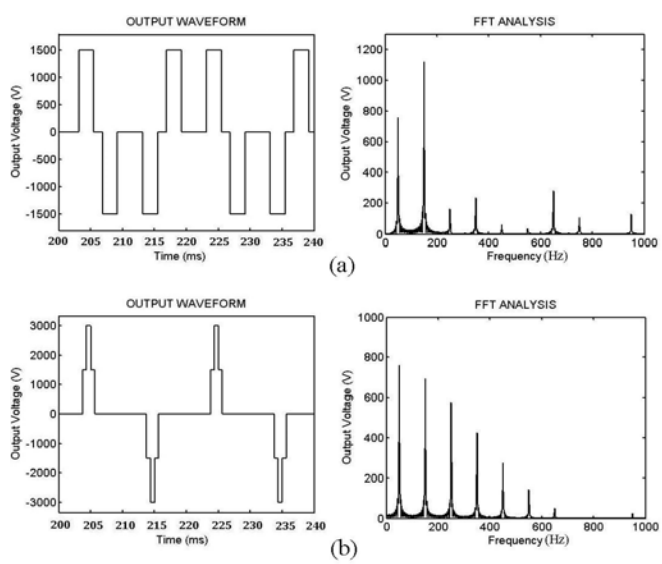

Fig. 11. Elimination of the seventeenth harmonic for *M=0.2*: (a) $\varphi = 4\pi/17$, (b) $\varphi = 16\pi/17$ (Simulation results).

VII. Experimental Results

To further verify the validity of the proposed method, a five-level diode-clamped inverter prototype is implemented with an input DC voltage of 600V ($V_{DC} = 150\ V$) and a 50Hz output frequency. The implemented prototype is shown in Fig. 12. The modulation index and switching angles of the following cases are the same as those employed in the above simulations. The results demonstrate the effectiveness of the proposed SHE method.

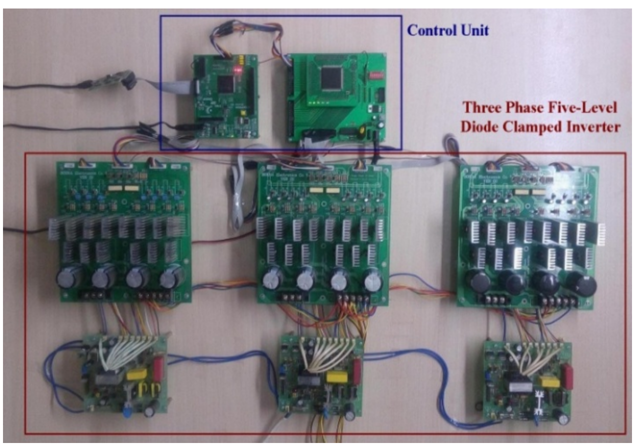

Fig. 12. Five-level diode clamped inverter prototype.

Case 1: Elimination of the third harmonic for $\boldsymbol{M = 0.8}$

As shown in Fig. 13, the output voltage waveform and its FFT spectrum are similar to those of Fig. 8. The base scales for for FFT analysis are 100 $\mathrm{V_{rms}/div}$ and $\mathrm{f = 62.5\ Hz/div}$.

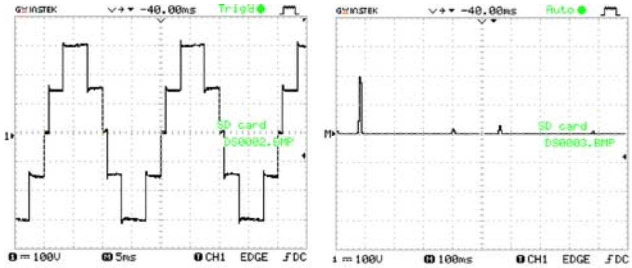

Fig. 13. Elimination of the third harmonic for *M=0.8*, using a phase difference of $\varphi = 2\pi/3$ (Experimental results).

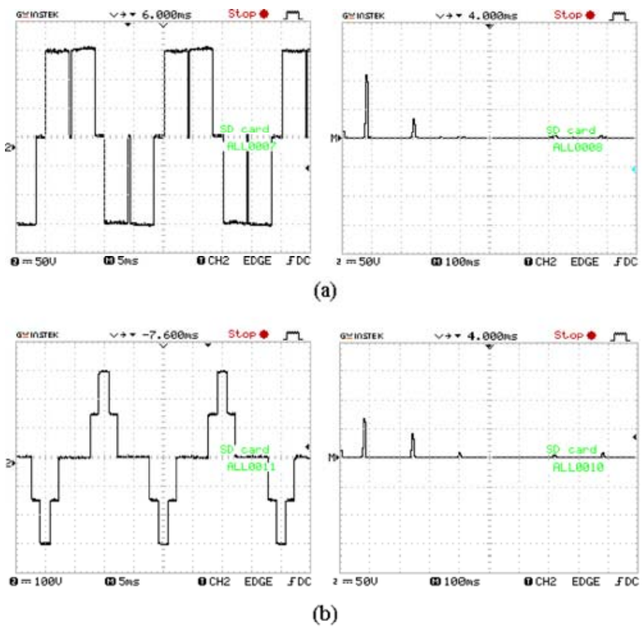

Fig. 14. Elimination of the seventh harmonic for *M=0.46*: (a) $\varphi = 4\pi/7$, (b) $\varphi = 6\pi/7$ (Experimental results).

Case 2: Elimination of the seventh harmonic for $\boldsymbol{M = 0.46}$

An output voltage waveform and its FFT spectrum for $\varphi = 4\pi/7$ are shown in Fig. 14(a). The voltage and frequency scales for the FFT analysis are 50 $\mathrm{V_{rms}/div}$ and $\mathrm{f = 62.5\ Hz/div}$, respectively. For $\varphi = 6\pi/7$, the results are shown in Fig 14(b) using 100 $\mathrm{V_{rms}/div}$ and $\mathrm{f = 62.5\ Hz/div}$ for the FFT base scales.

Case 3: Elimination of the thirteenth harmonic for $\boldsymbol{M = 0.9}$

Fig. 15 shows experimental results of this condition. The base scales for both of the FFT analyses are 100 $\mathrm{V_{rms}/div}$ and $\mathrm{f = 125\ Hz/div}$.

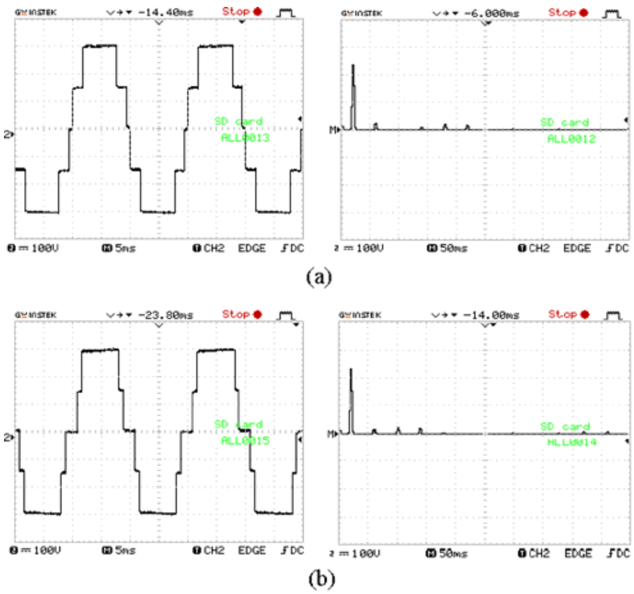

Fig. 15. Elimination of the thirteenth harmonic for *M=0.9*: (a) $\varphi = 10\pi/13$, (b) $\varphi = 12\pi/13$ (Experimental results).

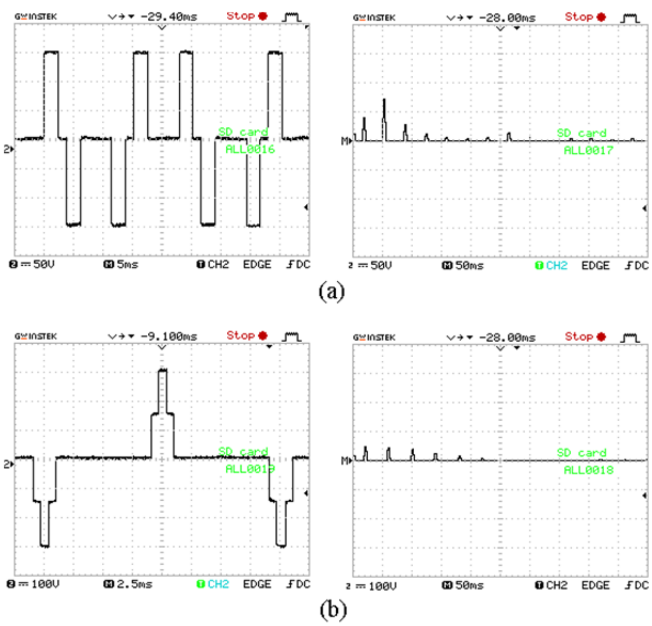

Fig. 16. Elimination of the seventeenth harmonic for *M=0.2*: (a) $\varphi = 4\pi/17$, (b) $\varphi = 16\pi/17$ (Experimental results).

Case 4: Elimination of the seventeenth harmonic for $\boldsymbol{M = 0.2}$

According to Fig. 16, the output waveform for $\varphi = 4\pi/17$ has only three levels and the base scale for its FFT analysis are $50\ \mathrm{V_{rms}/div}$ and $\mathrm{f = 62.5\ Hz/div}$. However, for $\varphi = 16\pi/17$, the output waveform has five levels and the base scale for its FFT analysis are $100\ \mathrm{V_{rms}/div}$ and $\mathrm{f = 62.5\ Hz/div}$.

The harmonic amplitudes in the experimental results are not exactly the same as those of the simulations because the switches and other components are not ideal. In addition, measurement errors make the experimental results not accurate. However, there exists a good adaptation between the simulation and experimental FFT spectrum for all cases.

VIII. Comparison of the Proposed Method with Other Methods

Numerical and Stochastic Optimization techniques need to have the initial values chosen, and only produce a single solution set. In addition, these techniques may get trapped in a local optima. On the other hand, the proposed method does not need have the initial values chosen, and is able to find all possible sets of solutions. Furthermore, this method can find the exact value of switching angles and can be implemented in real time as opposed to the aforementioned methods.

Finally, the proposed method in comparison with the analytical method proposed in [26] has more advantages. In [26], the modulation index intervals are calculated just for five-level staircase waveform. However, in the proposed method, a single unique set of equations can determine the modulation index intervals for both three-level and five-level waveforms. Consequently, it can introduce some solutions for the low values of the modulation indices by proposing three-level waveforms as opposed to the method in [26]. In other words, the proposed method extends the feasible modulation index range. In addition, for calculating the switching angles only one trigonometric operation is needed and the other arithmetic operations are sum and subtraction according to Fig. 6. Therefore, the proposed method is simpler than the one presented in [26] which is based on Chebyshev polynomials and Waring formulas.

IX. Conclusions

In this paper, a new expression for the SHE problem in five-level converters has been introduced, which is based on the phase shift of two identical quasi-square waveforms. Using this new expression leads to a single formulation for both three-level and five-level waveforms with two switching angles, which has the benefits of simplicity. An analytical procedure for determining the feasible modulation index intervals and all possible sets of solutions is presented. Furthermore, the exact values of the switching angles are calculated with a reduced computational burden. For the real-time implementation of this method in microcontrollers or DSPs, a simple flowchart is proposed which does not need a lookup table and can be easily implemented on various types of multilevel converters. Finally, the effectiveness and performance of the proposed method have been verified by various simulation and experimental results.

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