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Improving the Solution Range in Selective Harmonic Mitigation Pulse Width Modulation Technique for Cascaded Multilevel Converters

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Abstract

This paper proposes an improved low frequency Selective Harmonic Mitigation-PWM (SHM-PWM) technique. The proposed method well mitigates the low order harmonics of the output voltage up to 50th harmonics and satisfies the grid codes EN 50160 and CIGRE-WG 36–05. Using a modified criterion for the switching angles, the range of modulation index for the non-linear SHM equations is improved, without increasing the switching frequency of CHB converter. Due to low switching frequency of CHB converter and mitigating the harmonics of converter up to 50th order, and finding wider modulation index range, the size and cost of passive filters can significantly be reduced with the proposed technique. So, the proposed technique is more efficient than conventional SHM-PWM. To verify the effectiveness of the proposed method, a 7-level Cascaded H-bridge (CHB) converter is utilized for the study. Simulation and experimental results confirm the validity of above claims.

Key words: Multilevel converter, Selective Harmonic Mitigation, Cascaded H-bridge Inverter, Particle Swarm Optimization.

I. INTRODUCTION

Multilevel converters have gained a lot of attention in recent years due to their salient features such as lower stress across the semiconductors, lower common-mode voltage generation, lower harmonics in the output waveform, and lower EMI generation [1, 2]. In all of the multilevel structures, a stepwise voltage waveform is synthesized to reduce the total harmonic distortion (THD) and harmonic content. These converters are categorized as diode clamp converter, flying capacitor converter and cascaded H-bridge (CHB) converter. Among these topologies, the CHB has the highest modularity and can be easily scaled to different voltage and power levels; hence, it is selected for further study in this paper.

In the applications such as Flexible AC Transmission Systems (FACTS) [3], High Voltage DC lines (HVDC) [4], and electrical drives [5], it is important to achieve high efficiencies while the THD is minimum. This goal can be achieved through employing low frequency modulation techniques for the CHB converters. According to [6]-[18], selective harmonic elimination (SHE) and selective harmonic mitigation (SHM) and selective harmonic current mitigation PWM [31], [32] are the most prominent low frequency modulation techniques in the literature. The implementation of these methods in the CHB converter has been faced with the following challenges,

1) Due to a large number of non-linear equations, solving and finding the answers is not simple.

2) To limit the switching loss, the minimum number of switching transitions should be utilized to satisfy the grid codes.

In the SHM approach, instead of complete elimination of low order harmonics like the SHE method, the non-linear equations are solved to keep the low order harmonics smaller than the specified limits in the grid codes. Hence, as an advantage, the possibility of finding solutions for a wide range of modulation indices is higher. It is also worth mentioning that in conventional SHM, each cell has one switching transition in a quarter-cycle, but in selective harmonic mitigation-PWM (SHM-PWM) the number of switching transitions can be more than one. This feature helps to mitigate more number of harmonics without increasing the number of H-bridge cells in the CHB converter. Thus, the SHM-PWM is selected for further study and adoption in this article.

The SHM-PWM technique was first proposed in [12] for a CHB inverter. The switching frequency of each switch was limited to 750 Hz and the low order harmonics (lower than 50) were mitigated in a three-phase inverter. In [13], a closed loop implementation of the proposed idea in [14] was carried out and the results were satisfactory. The voltage of capacitors can be variable to increase the degrees of freedom[9], [15]-[20]. In this method, however, the inverter should be fed by variable DC sources which brings a lot of implementation difficulties.

A SHM-PWM approach based on equal DC-link voltages in the CHB inverter has been introduced in [21]. It uses nine switching angles in each quarter-cycle, so the switching frequency of each power switch is limited to 150 Hz. But, this method could only eliminate non-triplen harmonics up to 40^{th} term.

Conventionally, to obtain SHE-PWM and SHM-PWM equations for the CHB inverters, the switching angles of H-bridge cells are usually arranged sequentially. In [11], [22], however, a new idea for SHE-PWM technique has been presented which tries to eliminate the above criterion. The main goal of this idea is to increase the search space for the mathematical solver and to have a wider range of available solutions.

In [23], a combination of SHM and SHE modulation techniques was proposed, which a technique was used to produce appropriate waveforms for a four-leg three-level NPC inverter. In this work, by using SHM technique the non-triplen harmonics in the phase legs are mitigated. In addition, the fourth leg is controlled by the SHE to eliminate important low-order triplen harmonics. In [24], an inverter scheme of low frequency modulation index has been proposed for high power applications. Moreover, the SHE and SHM techniques were used in wide variety of applications [25].

In this paper, the idea of non-sequential switching angles is extended to the SHM-PWM modulation technique to improve the solution range. Much effort is devoted to mitigate non-triplen low order harmonics up to 50^{th} term just by nine switching angles in a quarter-cycle of the fundamental period. In other words, the switching frequency of the switches is limited to 150 Hz in a 7-level CHB inverter. Furthermore, the grid codes EN 50160 and CIGRE-WG 36–05 [21], [26], [27], are considered for the proposed SHM-PWM method. The validity of proposed method is verified by simulations and experiments on a 7-level CHB inverter.

II. Cascaded H-Bridge Structure

Among the multilevel converters, the CHB converter has a modular structure and needs the minimum number of components to synthesize the same number of voltage levels. As it is shown in Fig. 1, the CHB inverter is made of N series

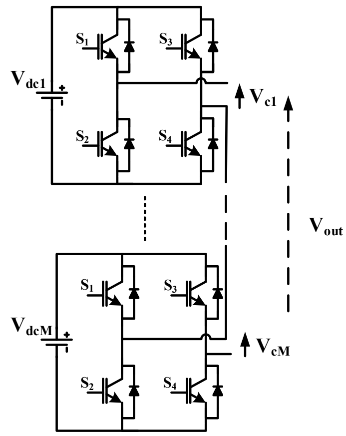


Fig. 1. Structure of CHB inverter.

TABLE I
THE MAIN SWITCHING STATES IN AN H-BRIDGE CELL

V_{ac}	S_1	S_2	S_3	S_4
$+V_{dc}$	1	0	0	1
0	1	0	1	0
0	0	1	0	1
$-V_{dc}$	0	1	1	0

connected H-Bridge cells. The CHB inverter tries to synthesize the desired AC output voltage (V_{out}) from the distinct DC voltages, i.e., V_{dc1}, V_{dc2}, ..., V_{dcM} . In the symmetric CHB structures, the voltages of all DC links are considered to be equal and this assumption is used in this article. Hereafter, the DC link voltages are assumed to be V_{dc}.

Each H-bridge cell in Fig. 1 can generate three different voltage levels, i.e., V_{dc}, 0, and $-V_{dc}$ in its AC terminals. Table I shows different switching states which can be used to synthesize a voltage level at the cell AC terminal. It is evident that the total number of voltage levels which can be synthesized at the inverter phase AC voltage is $2M+1$, where M is the number of H-Bridge cells.

III. Selective Harmonic Mitigation Modulation (SHM-PWM) for CHB Inverter

A. Basic equations of the SHM-PWM technique

In the SHM-PWM method, first a cycle of predefined voltage waveform is considered. Then, using the Fourier series analysis formula, the Fourier series coefficients of the voltage waveform is calculated. Next, the amplitude of each voltage harmonic is determined and is used in the non-equality equations of SHM-PWM, except the first harmonic which is used to control the amplitude of output voltage. In these equations, the upper limits are determined according to the selected grid codes which should be satisfied.

The general equation of the SHM-PWM method is shown in the following equation,

$$V(\omega t)=\sum_{n=1}^{\infty}(a_n cos(n\omega t)+b_n sin(n\omega t)) \quad (1)$$

where ω is the output frequency in radian. Also a_n and b_n are the coefficients of the Fourier series. In the predefined waveform in Fig. 2, a_n coefficients are eliminated in the output harmonic spectra, because of the quarter-wave symmetry; hence, The Fourier series expansion of the predefined waveform is simplified as:

$$V(\omega t)=\sum_{n=1}^{\infty} b_n sin(n\omega t) \quad (2)$$

where the b_n coefficients are

$$b_n=\begin{cases} 0 & n=even \\ \dfrac{4V_{dc}}{n\pi}\sum\limits_{\substack{i=1,\dots,m \\ j=1,\dots,n_i}} K_{ij}\cos n\theta_{ij} & n=odd \end{cases} \quad (3)$$

and K_{ij} is +1 if the i^{th} transition edge in Fig. 2 is rising and it is -1 if the transition edge is falling. The main purpose of the SHM-PWM method is to control the amplitude of fundamental harmonic and to mitigate the selected harmonics from the output voltage. The first coefficient in (3), i.e., b_1 controls the amplitude of fundamental harmonic (or modulation index ma) and it is determined by

$$b_1=\frac{4V_{dc}}{\pi}(\cos\theta_{11}-\cos\theta_{12}+\dots+ \cos\theta_{1n_1}+\dots+\cos\theta_{m1}+\dots+\cos\theta_{mn_m}) \quad (4)$$

$$m_a=(\cos\theta_{11}-\cos\theta_{12}+\dots+\cos\theta_{1n_1}+ \cos\theta_{21}\dots+\cos\theta_{m1}+\dots+\cos\theta_{mn_m}) \quad (5)$$

where m_a represents the modulation index and it can vary between 0 and m for a $2m+1$ level CHB converter. In this paper, a 7-level waveform is considered, which is shown in Fig. 3. The SHM-PWM equations are defined to satisfy the grid codes EN 50160 and CIGRE-WG 36–05 [15], [30], [31], which their limits are shown in Table II. As it is seen from Table II, the THD value must be restricted to 8%.

According to (2-5), the SHM-PWM equations for the predefined waveform in Fig. 3 are derived as:

$$\begin{cases} m_a=\cos\theta_{11}-\cos\theta_{12}+\cos\theta_{13}+\cos\theta_{21} \\ -\cos\theta_{22}+\cos\theta_{23}+\cos\theta_{31}-\cos\theta_{32}+\cos\theta_{33} \\ H_1=?\dfrac{4V_{dc}}{\pi}\ \cos\theta_{11}-\cos\theta_{12}+\cos\theta_{13}+\cos\theta_{21} \\ -\cos\theta_{22}+\cos\theta_{23}+\cos\theta_{31}-\cos\theta_{32}+\cos\theta_{33}) \\ \dfrac{4V_{dc}}{m\pi}\begin{pmatrix} \cos(m\theta_{11})-\cos(m\theta_{12})+\cos(m\theta_{13}) \\ +\cos(m\theta_{21})-\cos(m\theta_{22})+\cos(m\theta_{23}) \\ +\cos(m\theta_{31})-\cos(m\theta_{32})+\cos(m\theta_{33}) \end{pmatrix} \le |H_1| L_m \\ m\ ?5,?,?,?,?,19,?3,?5,29,31,35,37,41,43,47,49 \end{cases} \quad (6)$$

where H_1 is the amplitude of fundamental harmonic and L_m determines the upper limit of m^{th} harmonic according to the standards.

B. Proposed SHM-PWM algorithm

TABLE II.
QUALITY GRID CODS: EN 50160 AND CIGRE WG 36-05

Harmonic limits			
Non-triplen harmonics		Triplen harmonics	
Harmonic order, n	Voltage limits, Li	Harmonic order, n	Voltage limits, Li
5	6%	3	5%
7	5%	9	1.5%
11	3.5%	15	0.5%
13	3%	21	0.5%
17	2%	>21	0.2%
19	1.5%		
23	1.5%		
25	1.5%		
>25	0.2+32.5/n		
THD (40^{th})	8%		

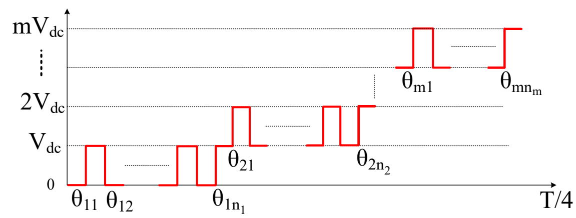


Fig. 2. Predefined voltage waveform in SHM-PWM

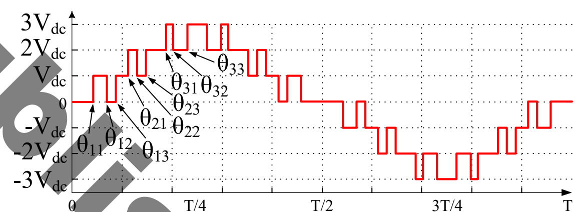


Fig. 3. A 7-level predefined voltage waveform in SHM-PWM

Generally, in a CHB converter, each level of the output waveform is generated by a specific cell. For example, in a $2m+1$ level converter like the one in Fig. 1 and for the predefined waveform in Fig. 2, the first, second and m^{th} H-bridge cells should contribute in the modulation in the phase intervals of ($\theta_{11}-\theta_{1n_1}$), ($\theta_{21}-\theta_{2n_2}$) and ($\theta_{m1}-\theta_{mn_m}$), respectively. This contribution can be shown by

$$0<\overbrace{\theta_{11}<\dots<\theta_{1n_1}}^{Cell\,1}<\dots<\overbrace{\theta_{m1}<\dots<\theta_{mn_m}}^{Cell\,m}<\frac{\pi}{2} \quad (7)$$

According to above rule, the 1st, 2nd, ..., m-1th cells will be turned on more than last cell, and therefore, their power losses will be unequal. As a result, this can reduce the reliability of the converter due to different power losses. So, the ON-time of the switches is a key factor to design the SHM-PWM technique in industrial applications.

Furthermore, solving the non-linear equations in (6) and finding the solutions over a wide range of modulation indices are not easy. In this paper, however, another rule is used for the sequence of switching which helps to get more degrees of freedom for solving the non-linear equations in the SHM-PWM method [23], as well as a better distribution of power loss. In the proposed rule, for a specific modulation index, three arbitrary switching angles are devoted to each H-bridge cell. As an example, the following arrangement may be applied for a specific modulation index,

$$\begin{gathered}0<\overbrace{\theta_{11}<\ldots<\theta_{1n_1}}^{Cell\,1}<\frac{\pi}{2}\\0<\overbrace{\theta_{21}<\ldots<\theta_{2n_2}}^{Cell\,2}<\frac{\pi}{2}\\\vdots\\0<\overbrace{\theta_{m1}<\ldots<\theta_{mn_m}}^{Cell\,m}<\frac{\pi}{2}\end{gathered} \tag{8}$$

According to (8), there is not any constraint for the H-bridge cells to involve in the modulation sequentially. In other words, each of the H-bridge cells can contribute to the transitions at n_i *(i=1,...,m)* arbitrary switching angles. This feature gives more relaxation to the mathematical solver in finding the solutions for the non-linear equations.

It is worth mentioning that for a specific modulation index, the command of the switching transitions can be changed between H-bridge cells in a regular way to equally distribute the power loss among them either in conventional or proposed method in this paper.

C. Solving the SHM-PWM equations

One of the key challenges associated with SHM-PWM is to solve the non-linear equations with the trigonometric terms. Many different methods have been proposed to solve the equations [28]-[30]. In this paper, particle swarm optimization (PSO) algorithm [21] is used to solve the equations.

The main difference between these techniques is in accuracy and speed of the optimization technique to find the solutions. The PSO technique uses the best local particles (solutions) in each iteration to guide other particles (solutions) in order to find the best global particle. Consequently, in each iteration the objective functions of the whole particles are checked to find the local and global best particles. The main equations which are used in the PSO technique is shown in the following equation,

$$v_j^{k+1}=wv_j^k+c_1r_1(pbest_j^k-x_j^k)+c_2r_2(gbest^k-x_j^k) \tag{9}$$

$$x_j^{k+1}=x_j^k+v_j^{k+1} \tag{10}$$

where w is a weighted factor which is determined by the iteration number. x and v are position and velocity of each particle, respectively. Moreover, in each iteration these parameters should be updated, while the best position of the local and global swarms are stored in the *pbest* and *gbest* parameters, respectively. The objective function defined as follows [19]:

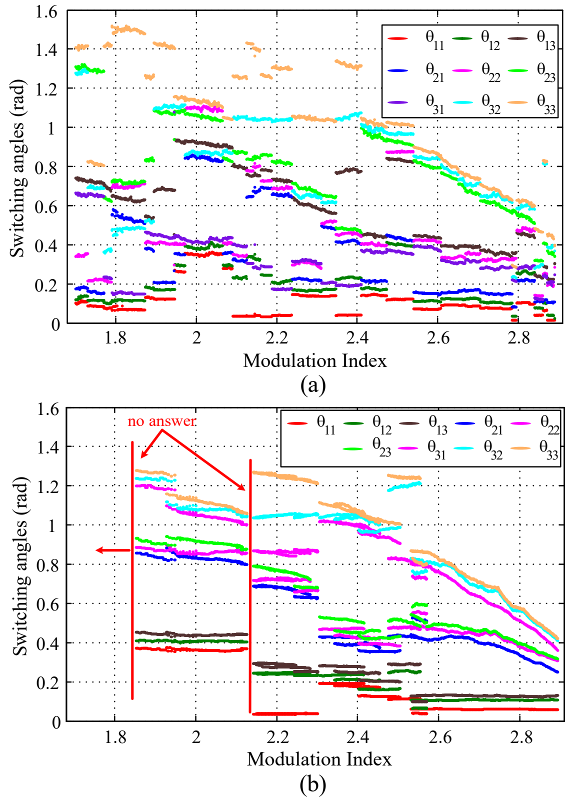


Fig. 4 Obtained solutions, (a) proposed SHM-PWM (b) conventional SHM-PWM.

$$OF(\theta_{11},\theta_{12},\ldots,\theta_{33})=\begin{cases}H_i\le L_i\left|H_1\right|\lambda, & i=5,7,11,\ldots,49\\H_1=\dfrac{4V_{dc}}{\pi}m_a\end{cases} \tag{11}$$

where H_i represents the i[th] objective function to be minimized and λ is a real factor lower than unity. If λ = *1*, the corresponding harmonic will be same as the defined limits in Table II.

For an interval of 1.7<m_a<2.89, the corresponding equations are solved and the results are shown in Fig. 4(a). It is seen that there are always answers for modulation indices between 1.7 and 2.89 while in the conventional approach, there is no answer in some modulation indices (e.g., in m_a= 2.13) or in m_a< 1.84. Fig. 4(b) shows the obtained answer sets by the conventional approach. It is also worth mentioning that one can find different solutions based on the proposed approach for a specific modulation index, which gives more flexibility in selection of answers from the view point of THD and power losses.

The solutions which are obtained and shown in Fig. 4 always meet the requirements of standard for both conventional and proposed technique. As long as a solution meet the requirements of standard for all harmonics up to 50[th] and THD, it could be considered as a solution. The main objective of this paper is to increase the solution range of SHM-PWM technique, not to control harmonic magnitudes less than conventional technique. So, for some harmonic

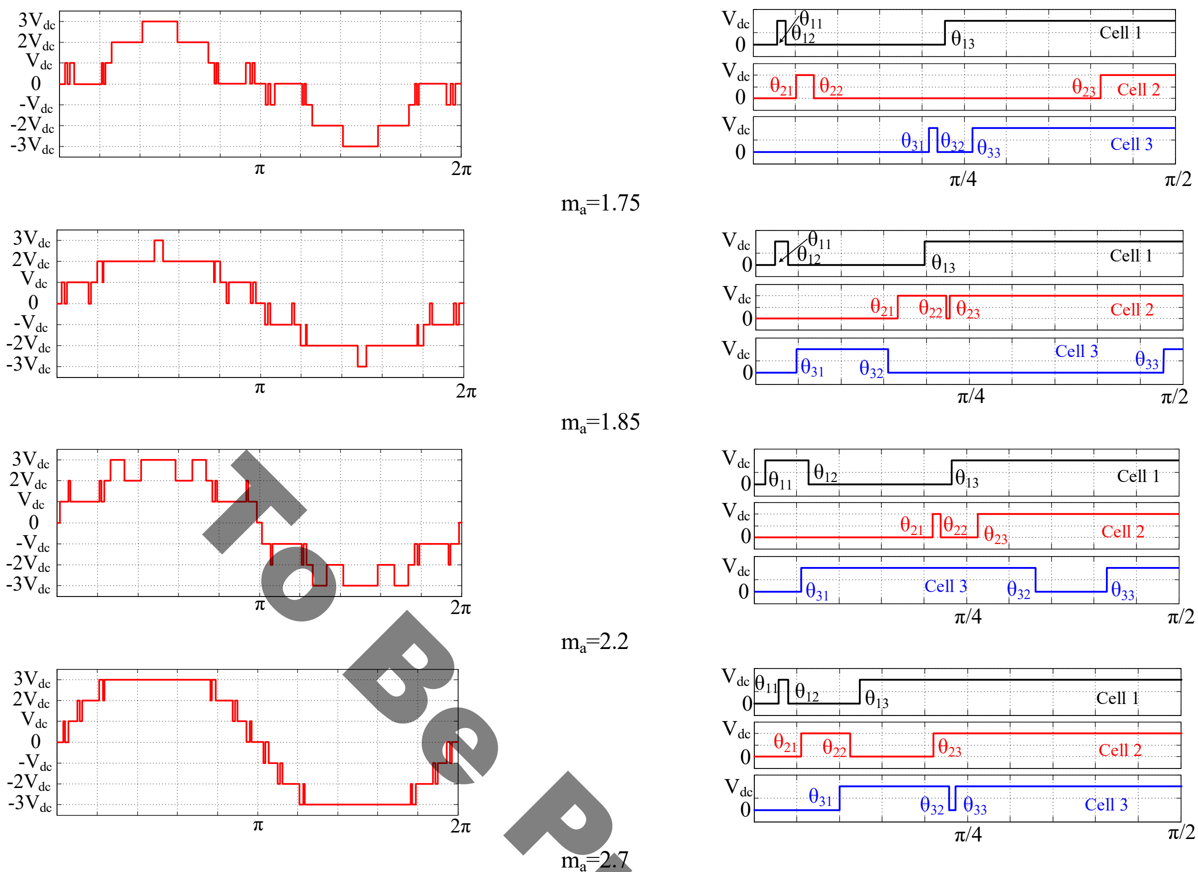


Fig. 5. Output phase voltage (left) and the corresponding voltage of each H-bridge cell (right) at different modulation indices.

TABLE III
SWITCHING ANGLES OF THE PROPOSED METHOD IN DIFFERENT MODULATION INDICES

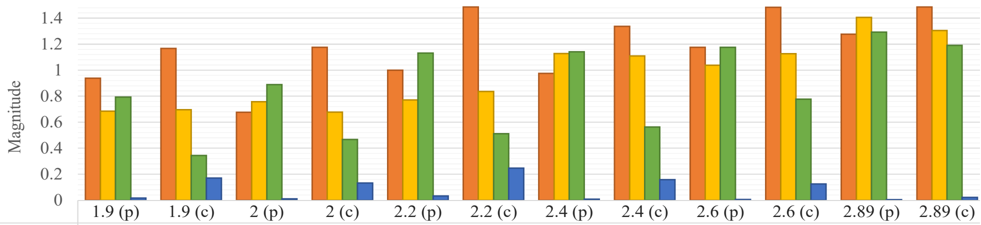
Modulation Index	θ_{11}	θ_{12}	θ_{13}	θ_{21}	θ_{22}	θ_{23}	θ_{31}	θ_{32}	θ_{33}
1.7	0.103366	0.121309	0.741659	0.176030	0.342558	1.311165	0.666244	1.280078	1.408369
1.75	0.087246	0.118919	0.714617	0.158518	0.225411	1.293509	0.654947	0.686776	0.816049
1.85	0.070206	0.117099	0.620986	0.52276	0.701065	0.713281	0.148911	0.486899	1.502086
1.9	0.121940	0.171233	0.680752	0.206532	0.409360	1.088634	0.451172	1.101606	1.427351
2.2	0.039570	0.200946	0.731467	0.660646	0.689968	0.827511	0.173996	1.03996	1.30489
2.5	0.122271	0.407316	0.841304	0.440196	0.879644	0.933045	0.368412	0.974253	1.017620
2.7	0.089698	0.125310	0.389775	0.173063	0.353182	0.659427	0.314969	0.716731	0.741867
2.745	0.076250	0.105631	0.351695	0.146971	0.320778	0.603458	0.275462	0.648835	0.684992
2.89	0.015	0.025432	0.303608	0.109799	0.286864	0.341842	0.248657	0.369296	0.397888

order of proposed technique, the harmonic magnitudes can be higher than conventional technique. However, because the harmonics meet the requirements of standard, there is no issue for them to be considered as a solution of SHM-PWM.

Extending the solution range of the SHM-PWM technique does not relate to the optimization technique that is used to solve (11). The only parameter that can extend the solution range is to increase constraints of the objective function. So, in this paper, by extending the switching angle constraints of the SHM-PWM, the range of the obtained solutions are extended as shown in Fig. 4 (a).

In Fig. 5, the output phase voltage (left) and the corresponding contribution of H-bridge cells (right) are shown for some specific modulation indices. As shown, the contribution of different H-bridge cells varies as a function of modulation index. The corresponding switching angles are shown in Table III.

The proposed method helps to achieve a better distribution of power loss among the H-bridge cells compared to the conventional approach. To verify this behavior, the ON time of different H-bridge cells in a quarter period is measured in radian and shown in Fig. 6. In addition, the ON time as a function of modulation index for both the conventional and the proposed approaches are also shown. As it can be seen, the variance of the proposed method is considerably smaller than the conventional SHE-PWM, especially for high modulation indices. So, the power losses of the proposed technique are distributed more evenly between the cells of the converter. So,



	1.9 (p)	1.9 (c)	2 (p)	2 (c)	2.2 (p)	2.2 (c)	2.4 (p)	2.4 (c)	2.6 (p)	2.6 (c)	2.89 (p)	2.89 (c)
Cell 1 (rad)	0.9385	1.1678	0.67611	1.17533	0.9999	1.4862	0.9742	1.3366	1.17539	1.48334	1.2768	1.48597
Cell 2 (rad)	0.68419	0.6956	0.7574	0.6782	0.7718	0.8369	1.1282	1.1084	1.0379	1.1263	1.40522	1.30551
Cell 3 (rad)	0.79308	0.34518	0.88926	0.46657	1.131	0.51075	1.14105	0.56277	1.175	0.77698	1.2927	1.1912
Variance	0.01628	0.17041	0.0115	0.13237	0.033	0.24659	0.00861	0.1581	0.00628	0.12473	0.00489	0.02207

Fig. 6. ON time of different cells as a function of modulation index in the proposed approach and the conventional SHE-PWM (p=proposed, c=conventional)

it can improve the reliability of the proposed technique than the conventional technique.

IV. Simulation and Experimental Results

In this paper, to verify the effectiveness of proposed SHM-PWM method, several simulations and experiments have been carried out on a 7-level CHB inverter. The MATLAB Simulink environment has been used to simulate the 7-level converter. The controlling system is shown in Fig. 6. Moreover, in practical implementation, a CORTEX M4 ARM processor is used as controller to implement the SHM-PWM algorithm. The parameters of the simulation and hardware prototype are shown in Table. IV. Moreover, the utilized hardware prototype in the experiments is shown in Fig. 8.

To show the validity of proposed method, the simulations are carried out based on the switching angles in table III for different modulation indices. The simulation results are shown in Table V. The MATLAB FFT toolbox is used to calculate the harmonics based on the data that is extracted from oscilloscope. The corresponding harmonic analysis of the experimental phase voltages are also given in Table VI. To compare experimental results of the proposed and conventional methods, the same tests are carried out and reported for both of them. As shown in Table VI, the proposed SHM-PWM method can mitigate low order harmonics of the output voltage up to 50th order. Furthermore, the triplen harmonics will automatically be eliminated from the harmonic spectra, when a 3-phase inverter is used. As shown, the proposed method successfully fulfills the grid code requirements, while the switching frequency is 150 Hz. Fig. 9 demonstrates the harmonic spectra of output waveform at m_a=2.7. Moreover, the waveforms of a practical implementation for modulation indices of m_a=1.9, 2.4 and 2.745 are shown in Fig. 10. The harmonic spectra and THD of the waveforms in Fig. 10 can be found in Table VI.

V. Discussion

The mitigation technique meets the voltage harmonic distortion limits of power quality standard instead of completely eliminating the low order harmonics of CHB voltage. So, the number of harmonics which can be mitigated in SHM-PWM is higher than 2k-1, where k is the number of switching transitions of single-phase CHB in each quarter of period. However, to find the exact number of harmonics that can be mitigated with k number of switching transitions, the optimization technique should be solved and checked with trial and error. As proven in this paper, with 9 switching transitions in each quarter of period, the conventional SHM-PWM technique has a very limited solution range ($1.84<m_a<2.89$ with some unsolved points). To solve this issue in the proposed technique, the constraints of switching angles for each cell of converter are improved. As a result, the solution range is increased to ($1.7<m_a<2.89$).

TABLE IV.
Simulink and Experimental Parameters.

Parameter	Symbol	Value
Number of H-Bridge modules	N	3
Nominal DC link voltage of the CHB inverter	V_{DC}	30 V
AC side voltage frequency	f	50 Hz
Switching frequency of inverter	f_{s-i}	150 Hz
Capacitance	C	4 mF
Power MOSFET	S	IRF540N

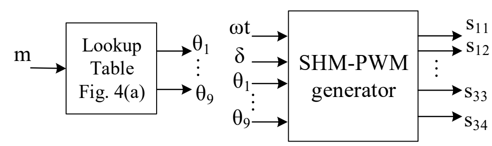


Fig. 7. The associated SHM-PWM generator.

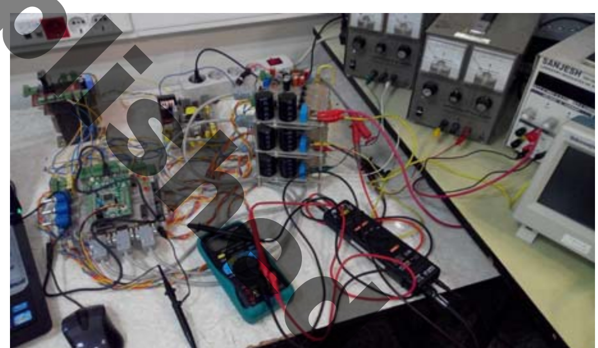


Fig. 8. Hardware prototype of the experimental investigation.

TABLE V.
THD AND HARMONIC COMPONENTS OF SIMULATION RESULTS IN THE PROPOSED AND CONVENTIONAL SHM-PWM METHOD.
(1. Maximum THD case, 2. Minimum THD case)

Harmonic order	Grid code limits,%	1.7[1]	1.75	1.85		1.9		2.2		2.5		2.7		2.745[2]		2.89	
		p	p	p	c	p	c	p	c	p	c	p	c	p	c	p	c
5	6	3.57	2.27	1.06	3.4	1.55	2.10	1.31	0.03	3.17	0.86	2.72	0.48	0.82	3.23	5.7	5.84
7	5	4.93	4.66	0.48	5	2.41	4.42	4.65	4.68	2.73	2.92	0.72	2.99	0.43	3.4	1.8	1.54
11	3.5	2.54	1.25	3.27	2.24	1.76	2.94	1.34	2.18	0.89	1.72	1	2.13	0.02	3.47	3.33	2.79
13	3	2.93	2.83	2.51	1.56	1.31	0.77	2.79	2.69	1.75	2.87	2.64	2.39	2.3	2.58	1.56	0.91
17	2	0.4	0.26	0.35	2	0.32	0.23	0.04	0.95	0.77	0.99	1	1.32	0.34	0.47	0.71	1.56
19	1.5	1.22	1.45	1.19	0.11	1.03	0.94	0.56	1.24	1.23	1.38	1.22	1.26	0.81	0.91	0.57	1.48
23	1.5	0.67	1.2	1.05	1.5	1.4	0.8	1.04	0.02	1.23	1.27	0.95	1.38	0.2	0.23	0.64	0.3
25	1.5	1.32	1.3	0.43	0.02	0.83	1.04	0.88	0.14	1.29	0.89	0.73	1.15	0.33	1.47	0.96	0.03
29	1.32	0.63	0.68	0.92	0.57	0.39	0.27	1.23	0.19	1.16	1.31	1	0.83	0.18	0.62	0.81	0.21
31	1.25	0.87	0.87	0.35	1.24	1.11	0.46	1.14	0.11	0.07	0.55	0.97	0.23	0.27	0.94	0.69	0.44
35	1.13	0.5	0.72	0.18	1.12	0.2	0.69	1.05	0.04	0.89	0.83	1.03	0.03	0.67	1.13	0.72	0.34
37	1.08	0.25	0.33	0.1	1.07	0.53	0.32	0.28	1.08	0.95	0.36	0.58	0.1	0.54	0.22	0.68	0.03
41	0.99	0.32	0.24	0.31	0.99	0.68	0.83	0.3	0.98	0.6	0.9	0.35	0.98	0.57	0.72	0.14	0.83
43	0.96	0.44	0.59	0.57	0.37	0.85	0.5	0.81	0.74	0.06	0.11	0.85	0.42	0.84	0.13	0.2	0.95
47	0.89	0.17	0.76	0.63	0.6	0.66	0.46	0.07	0.89	0.64	0.88	0.16	0.82	0.14	0.26	0.52	0.26
49	0.86	0.15	0.69	0.34	0.85	0.53	0.72	0.3	0.82	0.25	0.82	0.16	0.59	0.67	0.8	0.55	0.31
THD^{40th}	8	7.58	6.61	4.72	7.45	4.36	5.99	6.26	6.21	5.48	5.42	4.83	5.26	2.87	6.92	7.32	7.13
THD^{50th}		7.6	6.71	4.81	7.53	4.52	6.07	6.32	6.37	5.53	5.56	4.91	5.38	3.06	6.92	7.36	7.2
THD		14.29	12.91	12.57	14.95	11.75	11.56	11.34	11.73	10.77	10.12	10	9.69	8.66	9.62	9.71	9.51

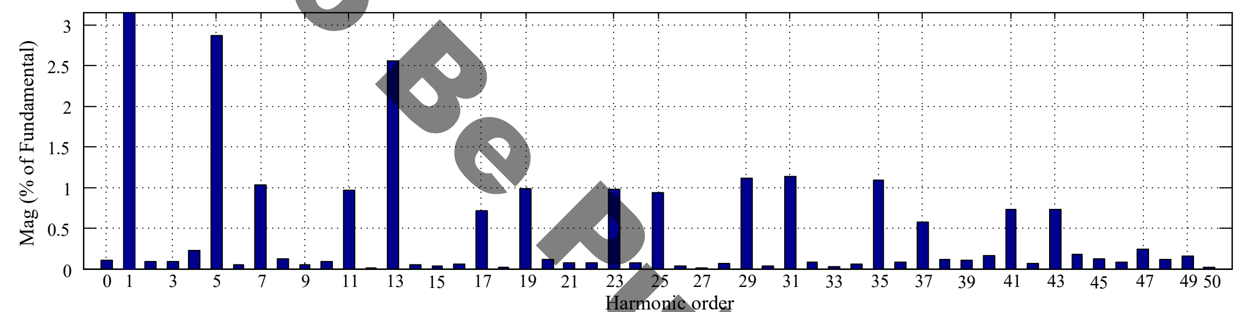


Fig. 10. Harmonic spectra of experimental results ma=2.7

VI. CONCLUSIONS

In this paper, the improved optimal low frequency SHM-PWM modulation technique was proposed. In the proposed method, the switching angle constraints are enlarged than conventional SHM-PWM for the H-bridge cells and this option gives more degrees of freedom to the math solver of non-linear equations. Hence, a larger set of answers can be found over a wider range of modulation indices. The proposed SHM-PWM method can successfully mitigate low order harmonics up to 50^{th} term, and it satisfies the grid codes EN 50160 and CIGRE-WG 36–05. As shown in this paper, by using the proposed switching constraint, the range of modulation index is improved near 30 percent, without increasing the switching frequency of CHB converter. Moreover, the proposed method shows a better distribution of power losses among the H-bridge cells.

TABLE VI.
THD AND HARMONIC COMPONENTS OF EXPERIMENTAL RESULTS IN THE PROPOSED SHM-PWM METHOD.

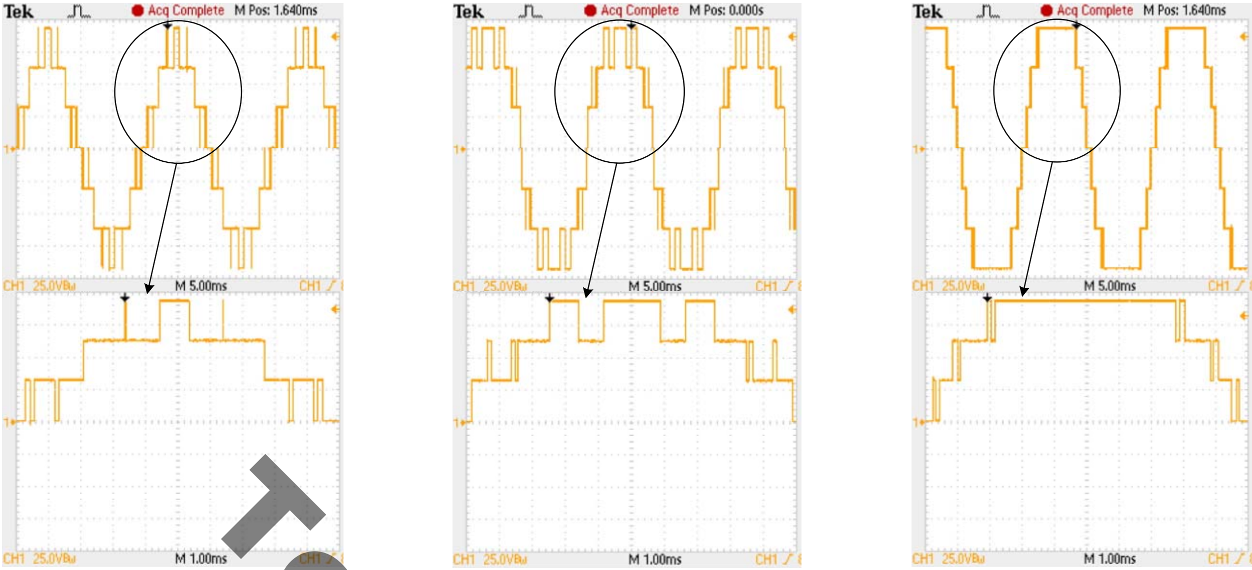
Harmonic Order	1.7	1.9	2.2	2.4	2.5	2.7	2.745
5	3.93	1.61	1.58	2.29	2.86	2.81	0.5
7	5	2.26	4.59	2.26	2.7	1.06	0.17
11	2.74	1.79	0.72	0.3	0.74	0.94	0.34
13	3	1.22	3	1.06	2.01	2.53	2.76
17	0.47	0.21	0.45	0.4	0.37	0.68	0.59
19	1.37	0.99	0.17	0.48	1.39	0.99	0.62
23	0.76	1.31	1.14	1.26	1.33	0.98	0.17
25	0.97	0.96	0.92	1.24	1.15	0.93	0.23
29	0.97	0.51	0.93	0.05	1.27	1.11	0.22
31	0.98	1.15	1.08	0.63	0.24	1.18	0.15
35	0.3	0.32	0.98	0.16	0.9	1.1	0.56
37	0.17	0.44	0.8	0.26	0.77	0.57	0.56
41	0.93	0.64	0.48	0.62	0.73	0.68	0.37
43	0.51	0.83	0.52	0.16	0.25	0.68	0.94
47	0.19	0.89	0.38	0.69	0.68	0.2	0.15
49	0.11	0.58	0.18	0.66	0.08	0.15	0.57
THD^{40th}	7.9	4.23	6.25	3.94	5.32	4.87	3.08
THD^{50th}	7.98	4.48	6.3	4.11	5.41	4.97	3.3
THD	14.62	11.89	11.42	8.88	10.65	9.81	8.65

REFERENCES

[1] J. Rodriguez, S. Bernet, B. Wu, J. O. Pontt and S. Kouro, "Multilevel Voltage-Source-Converter Topologies for Industrial Medium-Voltage Drives," IEEE Trans. Ind. Electron., vol. 54, no. 6, pp. 2930-2945, Dec. 2007.

[2] Fang Zheng Peng, Jih-Sheng Lai, J.W. McKeever, J. VanCoevering, "A multilevel voltage-source inverter with separate DC sources for static VAr generation," *IEEE Trans. Ind. Appl.*, vol.32, no.5, pp.1130-1138, 1996.

[3] Q. Song and W. Liu, "Control of a Cascade STATCOM With Star Configuration Under Unbalanced Conditions," IEEE Trans. Power Electron., vol. 24, no. 1, pp. 45-58, Jan.



(a) (b) (c)

Fig. 9. Experimental investigation of proposed method in synthesizing the SHM-PWM waveform, (a) m_a=1.9, (b) m_a=2.4, (c) m_a=2.745.

2009.

[4] N. Flourentzou, V. G. Agelidis and G. D. Demetriades, "VSC-Based HVDC Power Transmission Systems: An Overview," IEEE Trans. Power Electron., vol. 24, no. 3, pp. 592-602, March 2009.

[5] M. Hagiwara, K. Nishimura and H. Akagi, "A Medium-Voltage Motor Drive With a Modular Multilevel PWM Inverter," IEEE Trans. Power Electron., vol. 25, no. 7, pp. 1786-1799, July 2010.

[6] Y. Liu, H. Hong and A. Q. Huang, "Real-Time Calculation of Switching Angles Minimizing THD for Multilevel Inverters With Step Modulation," IEEE Trans. Ind. Electron., vol. 56, no. 2, pp. 285-293, Feb. 2009.

[7] L. M. Tolbert, J. N. Chiasson, Zhong Du and K. J. McKenzie, "Elimination of harmonics in a multilevel converter with nonequal DC sources," IEEE Trans. Ind. Appl., vol. 41, no. 1, pp. 75-82, Jan.-Feb. 2005.

[8] V. G. Agelidis, A. I. Balouktsis and C. Cossar, "On Attaining the Multiple Solutions of Selective Harmonic Elimination PWM Three-Level Waveforms Through Function Minimization," IEEE Trans. Ind. Electron., vol. 55, no. 3, pp. 996-1004, March 2008.

[9] M. K. Bakhshizadeh, M. Najjar, F. Blaabjerg and R. Sajadi, "Using variable DC sources in order to improve the voltage quality of a multilevel STATCOM with low frequency modulation," 2016 18th European Conference on Power Electronics and Applications (EPE'16 ECCE Europe), Karlsruhe, 2016.

[10] W. Fei, X. Du and B. Wu, "A Generalized Half-Wave Symmetry SHE-PWM Formulation for Multilevel Voltage Inverters," IEEE Trans. Ind. Electron., vol. 57, no. 9, pp. 3030-3038, Sept. 2010.

[11] M. Najjar, H. Iman-Eini and A. Moeini," Increasing the Range of Modulation Indices with the Polarities of Cells and Switching Constraint Reliefs for the Selective Harmonic Elimination Pulse Width Modulation Technique," *JPE*.

[12] L. G. Franquelo, J. Napoles, R. C. P. Guisado, J. I. Leon and M. A. Aguirre, "A Flexible Selective Harmonic Mitigation Technique to Meet Grid Codes in Three-Level PWM Converters," Trans. Ind. Electron., vol. 54, no. 6, pp. 3022-3029, Dec. 2007.

[13] J. Napoles, R. Portillo, J.I. Leon, M.A. Aguirre, L.G Franquelo, "Implementation of a closed loop SHMPWM technique for three level converters," 34th Annual Conference of IEEE. Industrial Electronics, 2008. IECON 2008, vol., no., pp.3260-3265, 10-13 Nov. 2008.

[14] M. S. A. Dahidah, G. S. Konstantinou and V. G. Agelidis, "Selective harmonic elimination pulse-width modulation seven-level cascaded H-bridge converter with optimised DC voltage levels," IET Power Electronics, vol. 5, no. 6, pp. 852-862, July 2012.

[15] N. Ghasemi, F.Zare, A.A. Boora, A. Ghosh, C. Langton, F. Blaabjerg, "Harmonic elimination technique for a single-phase multilevel converter with unequal DC link voltage levels," IET Power Electronics, vol.5, no.8, pp.1418-1429, September 2012.

[16] M. Najjar, A. Moeini, M. K. Bakhshizadeh, F. Blaabjerg and S. Farhangi, "Optimal Selective Harmonic Mitigation Technique on Variable DC Link Cascaded H-bridge Converter to Meet Power Quality Standards," IEEE *J Emerg Sel Top Power Electron*, vol. 4, no. 3, pp. 1107-1116, Sept. 2016.

[17] A. Moeini, H. Iman-Eini, and M. Najjar, "Non-equal DC link voltages in a cascaded H-Bridge with a selective harmonic mitigation-PWM technique based on the fundamental switching frequency," *Journal of Power Electronics*, Vol. 17, No. 1, pp. 106-114, Jan. 2017.

[18] N.Farokhnia, S.H. Fathi, N. Yousef poor, M.K. Bakhshizadeh, "Minimization of total harmonic distortion in a cascaded multilevel inverter by regulating voltages of dc sources," IET Power Electron., vol. 5, no. 1, pp. 106-114, January 2012.

[19] A. Moeini, H. Iman-Eini, M. Bakhshizadeh, "Selective harmonic mitigation-pulse-width modulation technique with variable DC-link voltages in single and three-phase cascaded H-bridge inverters," IET Power Electronics, vol.7, no.4, pp.924-932, April 2014.

[20] J. Napoles, A.J. Watson, J.J. Padilla, J.I. Leon, L.G. Franquelo, P.W. Wheeler, M.A. Aguirre, "Selective Harmonic Mitigation Technique for Cascaded H-Bridge Converters With Nonequal DC Link Voltages," IEEE

Trans. Ind. Electron., vol.60, no.5, pp.1963-1971, May 2013.

[21] A. Marzoughi, H. Imaneini, A. Moeini, “An optimal selective harmonic mitigation technique for high power converters”, International Journal of Electrical Power & Energy Systems, vol. 49, pp.34-39, July.2013.

[22] J. Vassallo, J.C. Clare, P.W. Wheeler, "A power-equalized harmonic-elimination scheme for utility-connected cascaded H-bridge multilevel converters," The 29th Annual Conference. Industrial Electronics Society, 2003, vol.2, no., pp.1185-1190, 2-6 Nov. 2003.

[23] M. Sharifzadeh, H. Vahedi, A. Sheikholeslami, P.-A. Labbe, K. Al-Haddad, "Hybrid SHM–SHE Modulation Technique for a Four-Leg NPC Inverter With DC Capacitor Self-Voltage Balancing," ," IEEE Trans. Ind. Electron., vol.62, no.8, pp.4890-4899, Aug. 2015.

[24] C. Jiankun, X. Shaojun, X. Jinming, "Research on a high power inverter with low frequency modulation index by selective harmonic mitigation technique," 9th International Conference. Power Electronics and ECCE Asia (ICPE-ECCE Asia), vol., no., pp.2409-2414, 1-5 June 2015.

[25] J. Napoles, J.I. Leon, R. Portillo, L.G. Franquelo, M.A. Aguirre, "Selective Harmonic Mitigation Technique for High-Power Converters," IEEE Trans. Ind. Electron., vol.57, no.7, pp.2315-2323, July 2010.

[26] EN50160.Voltage characteristics of electricity supplied by public distribution systems; 2004.

[27] CIGRE JWG C4.07, Power quality indices and objectives; 2004.

[28] A. Kavousi, B. Vahidi, R. Salehi, M. Bakhshizadeh, N. Farokhnia, S.S. Fathi, "Application of the Bee Algorithm for Selective Harmonic Elimination Strategy in Multilevel Inverters," IEEE Trans. Power Electron., vol.27, no.4, pp.1689-1696, April 2012.

[29] B. Ozpineci, L.M. Tolbert, J.N. Chiasson, "Harmonic optimization of multilevel converters using genetic algorithms," IEEE Power Electron Lett., vol.3, no.3, pp.92-95, Sept. 2005.

[30] R.N. Ray, D. Chatterjee, S.K.Goswami, "Harmonics elimination in a multilevel inverter using the particle swarm optimisation technique*," IET Power Electronics*, vol.2, no.6, pp.646-652, Nov. 2009.

[31] A. Moeini; H. Zhao; S. Wang, "Improve Control to Output Dynamic Response and Extend Modulation Index Range with Hybrid Selective Harmonic Current Mitigation-PWM and Phase-shift PWM for Four-Quadrant Cascaded H-Bridge Converters," IEEE Trans. Ind. Electron., vol.PP, no.99, pp.1-1.

[32] A. Moeini; H. Zhao; S. Wang, "A Current Reference based Selective Harmonic Current Mitigation PWM Technique to Improve the Performance of Cascaded H-bridge Multilevel Active Rectifiers," IEEE Trans. Ind. Electron., vol.PP, no.99, pp.1-1.

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